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The open story of the magnetic fluxes

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Abstract: We discuss the effects of *oblique* internal magnetic fields on the spectrum of type I superstrings compactified on tori. In particular we derive general formulae for the magnetic shifts and multiplicities of open strings connecting D9-branes with arbitrary magnetic fluxes. We discuss the flux induced potential and offer an interpretation of the stabilization of R-R moduli associated to deformations of the complex structure of $\mathbf{T}^6$ in terms of non-derivative mixings with NS-NS moduli. Finally we briefly comment on how to extract other low energy couplings and generalize our results to toroidal orbifolds and other configurations governed by rational conformal field theories on the worldsheet.

Keywords: p-branes, D-branes, String Duality, Intersecting branes models.

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Contents

1. Introduction

Constant electromagnetic fields coupled to the ends of open strings lead to many interesting effects that can be treated exactly in string theory [1 − 7] and have far reaching consequences in phenomenologically viable vacuum configurations [8 − 13]. The presence of constant abelian magnetic fields changes the open string boundary conditions in such a way that the oscillator modes of charged strings get shifted and the zero modes of neutral strings (KK momenta and windings) get reshuffled [14, 15].

So far most of the analyses focussed on "parallel" magnetic fields F_a on "factorizable" tori $\mathbf{T}^d = \mathbf{T}^2 \times \cdots \mathbf{T}^2$ or toroidal orbifolds that can be T-dualized to a single kind of branes intersecting at angles, for reviews see [17 − 20]. The boundary conditions are coded in the mutually commuting $d \times d$ matrices

$$R_a = (G - F_a)(G + F_a)^{-1}$$

satisfying the condition $R^t G R = G$. As shown in [21] for a toroidal compactification of type I strings to $D = 4$, properly turning on *oblique* but yet abelian magnetic fields, such that $[R_a, R_b] \neq 0$, in "non-factorizable" subtori with common directions may lead to closed string moduli stabilization while keeping some residual supersymmetry. These configurations are T-dual to bound states of several different kinds of intersecting D-branes [7, 22 − 26].

Here, we present a detailed analysis of the resulting open string spectrum and discuss the resulting potential for the closed string moduli fields, as well as their mixing and stabilization. For simplicity, as in [21], we work at the origin of the open string moduli space. While including the effect of Wilson line moduli, which are neutral with respect to the magnetic fields, should not pose any problem, turning on VEV's for charged open string states that contribute to D-terms might jeopardize the proposed stabilization mechanism for the closed string moduli associated to deformations of the Kähler structure[1]. An exact non-linear analysis of this and related effects is beyond the scope of the present investigation, since in principle it would require the knowledge of the non-abelian extension of the Born-Infeld (BI) action. One can however envisage the possibility of building models without charged singlets that should not suffer from this drawback since charged non-singlet states also contribute to non-abelian D-terms, that do not admit FI terms. Moreover F-terms, most notably Yukawa couplings, can prevent charged states from getting a VEV at zero cost in energy. Though very interesting if not crucial for the proposed stabilization mechanism, a serious study of this problem requires a deeper and not yet available understanding of non-abelian twist field correlators on the disk.

In order to fix our notation and describe some of the relevant issues in a simpler setting we start by reviewing the very familiar case of a 2-dimensional torus in section 2 and the less familiar case of a 3-dimensional torus, where *oblique* magnetic fields first show up, in section 3. In both these cases, however, turning on magnetic fields breaks all supersymmetries and prevents the existence of any stable configuration at finite internal volume even if one introduces exotic Ω-planes. We discuss the more interesting case of a 4-dimensional torus in section 4 where we derive a very general formula for the mode shifts in the presence of arbitrary magnetic fields and argue that all closed string moduli, except the overall volume and the dilaton, can be stabilized in a supersymmetric fashion, modulo mixing with neutral and charged open string singlets mentioned above. Scale invariance of the (anti) self-duality conditions should be held responsible for the arbitrariness of the overall volume. In section 5, we sketch how to embed these neat results into a 5-dimensional torus. All this is meant to be propaedeutic to the analysis of the phenomenologically relevant case of a 6-dimensional torus studied in section 6. After some general considerations, we restrict our attention to supersymmetric configurations and describe how to compute in full generality the magnetic shifts for *oblique* yet supersymmetry preserving magnetic fields. We also clarify the mechanism of stabilization of R-R moduli associated to deformations of the complex structure in terms of non-derivative mixings with NS-NS moduli. To this end we adapt and generalize the closed string scattering amplitudes on D-branes computed in [27, 28] to our case and derive similar formulae for scattering on Ω-planes [29]. We present our conclusions and perspectives for future investigation in section 7. In particular, we sketch how to extract tree-level and one-loop gauge couplings from (simple modifications of) the results in the previous sections. Further details can be found in [30]. We also discuss some

[1]We thank the referee among others to pointing this out to us.

generalizations to toroidal orbifolds and to vacuum configurations described by rational conformal field theories on the worldsheet. Very much as in [6], we argue that magnetic fluxes lead to twisted representations of the relevant chiral algebra.

2. Two-dimensional magnetized torus, yet again

A magnetic field on $\mathbf{T}^2$ is necessarily proportional to the volume (Kähler) form $J = dx^1 dx^2 = d\bar{z}dz/2i$. In complex coordinates

$$F = F_{12} dx^1 dx^2 = i f dz d\bar{z}/2$$

and the corresponding magnetic rotation matrix reads

$$R = (\mathbb{1} - H)(\mathbb{1} + H)^{-1} = \mathrm{diag}(e^{2i\beta}, e^{-2i\beta}) \tag{2.1}$$

where $H = 2\pi\alpha' F/Vol(\mathbf{T}^2)$ denote frame components and

$$\tan(\beta) = h \qquad \text{so that} \qquad \tan(2\beta) = \frac{2h}{1-h^2}\,. \tag{2.2}$$

In a real basis, one would have instead

$$R = \begin{pmatrix} \frac{1-h^2}{1+h^2} & -\frac{2h}{1+h^2} \\ \frac{2h}{1+h^2} & \frac{1-h^2}{1+h^2} \end{pmatrix}. \tag{2.3}$$

Consistency at the quantum level requires Dirac flux quantization

$$h = \frac{m}{n}\frac{\alpha'}{r_1 r_2} \tag{2.4}$$

where the magnetic monopole number (m) and the "wrapping" number (n) are integers, in the absence of a (quantized) NS-NS antisymmetric tensor background [31 – 33]. Otherwise m gets shifted by Bn [14, 15].

Thanks to the abelian nature of the rotation group in two dimensions, when several abelian magnetic fields are present the magnetic shifts of the modes of open strings connecting the stacks a (with flux h_a) and b (with flux h_b) are given by [2, 4, 3]

$$\epsilon_{ab} = \frac{\beta_{ab}}{\pi} = \frac{1}{\pi}[\arctan(q_a h_a) + \arctan(q_b h_b)] \tag{2.5}$$

where $q_a, q_b = \pm 1$ take care of possible images under orientation reversal, i.e. worldsheet parity Ω [14]. Let us stress that the magnetic rotation

$$R_{ab} = R^{q_a}(h_a) R^{q_b}(h_b) = R(q_a h_a + q_b h_b) = R(2\beta_{ab}) \tag{2.6}$$

appearing in the boundary state [34], acts by *twice* the amount determining the magnetic shift, $\beta_{ab} = \pi\epsilon_{ab}$.

In order to properly count the multiplicities of charged strings, one has to rescale N_a to $N_a n_a$ as a consequence of the multiple wrapping. In general, the target space coordinates X^i do not necessarily coincide with the world-volume coordinates σ^α but are rather linear functions of the latter. The jacobian matrix $W_\alpha{}^i = \partial X^i/\partial\sigma^\alpha$ of the linear transformation mapping the world-volume Σ^d onto the target space $\mathbf{T}^d$ must have integer entries. As a result, $W = \det(W_\alpha{}^i) = n$, is an integer, e.g. $W = 1$ for single wrapping $X^i = \delta^i_\alpha\sigma^\alpha$. The induced world-volume metric is thus $\mathcal{G}_{\alpha\beta} = W_\alpha{}^i W_\beta{}^j G_{ij}$. Clearly there is a separate wrapping matrix per each stack of (magnetized) branes $W_\alpha{}^i_{(a)}$ labeled by the index a and $n_a = \det(W_a)$.

Open strings with $q_b = -q_a$ and $a \neq b$, belong to the representation (N_a, N_b^*). For $q_a = q_b$, which is possible only in the unoriented case, one gets (N_a, N_b). In both cases the degeneracy of the Landau levels

$$I_{ab} = q_a m_a n_b + q_b m_b n_a \tag{2.7}$$

coincides with the first Chern number $c_1(\mathcal{E}_{ab})$ of the abelian gauge bundle $\mathcal{E}_{ab} = \mathcal{E}_a^{q_a} \otimes \mathcal{E}_b^{q_b}$. In the superstring case, I_{ab} counts the number of charged massless fermions, but the spectrum contains open string tachyons whenever $h_a \neq 0$ for some a.

Neutral strings starting and ending on the same stack $a = b$ of branes and thus with $q_b = -q_a$ have integer modes but carry "rescaled" KK momenta $p_i = k_i/nr_i\sqrt{1+h^2}$ with $i = 1, 2$. In the unoriented case, doubly charged strings, that start on a stack a and end on its image $b = \tilde{a}$ under Ω with $q_{\tilde{a}} = q_a$ are counted by the first Chern number of the doubly tensored gauge bundle $\mathcal{E}_a^{2q_a}$, i.e. $I_{aa} = c_1(\mathcal{E}_a^{2q_a}) = 2c_1(\mathcal{E}_a^{q_a})$.

T-duality along the $y = x^2$ axis maps D2-branes filling $\mathbf{T}^2$ into D1-branes wrapped around a direction forming an angle $\beta = \arctan(m\tilde{r}_2/nr_1)$ with the $x = x^1$ axis [35]. The magnetic shifts (up to a factor of π) are T-dual to the intersection angles. In the T-dual picture, I_{ab} counts the number of "algebraic" intersections [20], while the magnetically rescaled momenta map to momenta along the multiply wrapped D1-branes and windings in the orthogonal direction [18, 15].

Very much as for the heterotic string on twisted tori [36], internal magnetic fluxes can generate a potential for the closed string NS-NS and R-R moduli, G_{ij} and C_{ij}, as well as for the dilaton ϕ and the open string Wilson line moduli A_i^a [37−39]. The crucial difference is that for (unoriented) open strings the potential appears at half closed string loop order. The Born-Infeld contribution of the D-branes (disk) and the (negative) tension contribution of the Ω-plane (projective plane) [40, 41] combine to give

$$V(\phi, G_{ij}; m_a, n_a) = \tau e^{-\phi}\left(2\sum_a N_a\sqrt{\det(\mathcal{G}_a + \mathcal{F}_a)} \mp 32\sqrt{\det(G)}\right), \tag{2.8}$$

where $\mathcal{G}^a_{\alpha\beta}$, as discussed above, is the induced metric on the a^{th} stack of branes, $\mathcal{F}^a_{\alpha\beta}$ is the world-volume magnetic field

$$\mathcal{F}^a_{\alpha\beta} = \partial_\alpha A^a_\beta - \partial_\beta A^a_\alpha = W_\alpha^{(a)\,i} W_\beta^{(a)\,j} F^a_{ij}\,, \tag{2.9}$$

and τ is the constant (moduli independent) part of the brane tension. The factor of 2 takes care of image branes and we have allowed for exotic Ω-planes with positive tension.

Neglecting open string moduli and the WZ coupling to the R-R moduli, V turns out to depend only on the positive-definite overall volume $\omega = \sqrt{\det(G)}$ and the dilaton ϕ but not on the complex structure moduli. Independently of the sign of the Ω-plane tension,

$$V(\phi, \omega; m_a, n_a) = \tau e^{-\phi} \left(2 \sum_a N_a \sqrt{n_a^2 \omega^2 + m_a^2} \mp 32|\omega| \right) \tag{2.10}$$

displays a runaway behavior to infinity for ϕ. ω runs to infinite, respectively zero, value for standard, respectively, exotic (anti) Ω-planes. The R-R tadpole condition, $2\sum_a n_a N_a = \pm 32$, can be satisfied for standard as well as exotic Ω-planes [32, 42], depending on the signs of n_a.

We will not dwelve any further on the issue of moduli stabilization on $\mathbf{T}^2$ since magnetic fluxes break supersymmetry anyway and generate tachyons in the open superstring spectrum. Supersymmetric configurations could be found on product of tori[2] $\prod_I \mathbf{T}^2_{(I)}$ and orbifolds thereof, which have been the main subject of investigation so far. The possibility of moduli stabilization by means of magnetic fluxes does not improve in these cases but would require, for instance, turning on RR 3-form fluxes [37, 38] or other non-perturbative effects. The above formulae for the magnetic shifts, multiplicities and zero-modes can be straightforwardly generalized and the same is true for the Born-Infeld and WZ couplings [40, 41]. Consistent action of the orbifold group, that drastically reduces the number of untwisted closed moduli but unfortunately introduces many new twisted moduli, usually restricts the allowed open and closed string fluxes. However the only NS-NS moduli that appear in the potential generated by untwisted magnetic fields are the volume moduli ω_I surviving the orbifold projections that cannot be stabilized without the help of subtle non-perturbative effects.

3. Three-dimensional magnetized torus

In three dimensions a magnetic field has three independent components

$$F = F_{xy} dxdy + F_{yz} dydz + F_{zx} dzdx \,.$$

Any proper rotation has at least one unit eigenvalue and two complex conjugate ones of unit norm. In order to work with standard orthogonal matrices one has to switch to the 'frame' components where the flat metric is Euclidean $\hat{G}_{\hat{i}\hat{j}} = E^i_{\hat{i}} E^j_{\hat{j}} G_{ij} = \delta_{\hat{i}\hat{j}}$ but the cell of the torus is generally skew. Matrix manipulations are safer in this basis as R becomes a rotation matrix

$$R = (1 - H)(1 + H)^{-1} \,,$$

where $H_{\hat{i}\hat{j}} = E^i_{\hat{i}} E^j_{\hat{j}} F_{ij}$. By elementary physics the unmagnetized direction points along the magnetic field $\vec{H} = (H_{\hat{y}\hat{z}}, H_{\hat{z}\hat{x}}, H_{\hat{x}\hat{y}})$. Setting $h = |\vec{H}|$, the modes of the coordinates orthogonal to $\vec{H}$ of a singly charged open string, starting on a magnetic brane and ending on an unmagnetized one, are shifted by

$$\epsilon = \frac{\beta}{\pi} \qquad \text{with} \qquad \beta = \arctan(qh) \,. \tag{3.1}$$

[2] See however [25].

The situation is slightly more involved for a string starting on a magnetized brane labeled by a and ending on another one labeled by b.

As anticipated in the introduction and suggested by boundary state considerations [34], one has to diagonalize the rotation matrix $R_{ab} = R_a^{q_a} R_b^{q_b}$, where R_a, (R_b) is a rotation around $\vec{H}_a$ $(\vec{H}_b)$ by an angle $2\beta_a$ $(2\beta_b)$. The eigenvector corresponding to the unit eigenvalue identifies the unmagnetized direction $\vec{u}_{ab}$, while the two orthogonal directions generically suffer a non-trivial action of R_{ab}. For practical reasons, instead of working in the 3-dimensional vector representation of SO(3) $\approx$ SU(2), it turns out to be much more rewarding to work in the spin 1/2 representation and exploit the remarkable properties of Pauli matrices. A magnetic rotation can be written as

$$R^q_{(1/2)} = \cos(\beta) - iq\hat{H}\cdot\sigma\sin(\beta) \tag{3.2}$$

where $\hat{H} = \vec{H}/|\vec{H}|$ is the unit vector in the direction of the magnetic field. Let us stress once more that the argument of the trigonometric functions is correctly $\beta = 2\beta/2$. Denoting for short $\cos(\beta_{a/b})$, respectively $\sin(\beta_{a/b})$, by $c_{a/b}$, respectively $s_{a/b}$, and combining the two rotations yields

$$R_{ab} = (c_a c_b - q_a q_b s_a s_b \hat{H}_a\cdot\hat{H}_b) - i(q_a s_a c_b \hat{H}_a + q_b c_a s_b \hat{H}_b + q_a q_b s_a s_b \hat{H}_a\times\hat{H}_b)\cdot\sigma \tag{3.3}$$

that exposes the 'unmagnetized' direction

$$\vec{u}_{ab} = q_a s_a c_b \hat{H}_a + q_b c_a s_b \hat{H}_b + q_a q_b s_a s_b \hat{H}_a\times\hat{H}_b \tag{3.4}$$

and determines the magnetic shifts $\epsilon_{ab} = \beta_{ab}/\pi$ via

$$\cos(\beta_{ab}) = c_a c_b - q_a q_b s_a s_b \hat{H}_a\cdot\hat{H}_b\,. \tag{3.5}$$

Though obvious, let us stress once more that $\beta_{ab} \neq \beta_a \pm \beta_b$ in general, contrarily to the two-dimensional as bad as any factorizable case. Another amusing feature is the orientation of the 'unmagnetized' direction $\vec{u}_{ab}$ with respect to the generators of the fundamental cell. Due to Dirac quantization condition on H_a, in each sector of the spectrum one can redefine the fundamental cell and choose $\vec{u}_{ab}$ to be one of the generators thereof. The generalized KK momenta carried by neutral strings turn out then to be rather involved combinations of momenta and windings. These are not so surprising features of the non-abelian structure of the rotation group.

Unfortunately, there is no non-trivial way to preserve supersymmetry and, as in the two dimensional case, one can neither avoid tachyons when magnetic fields are turned on, nor stabilize the vacuum perturbatively.

4. Four-dimensional magnetized torus

Magnetized four-dimensional tori display some remarkable features. This is the lowest dimensional case where non-trivial supersymmetric configurations first appear. Moreover thanks to the non-simplicity of the rotation group, SO(4) = SU(2)$_L$ $\times$ SU(2)$_R$, one can compactly analyze the problem for arbitrary magnetic fields. To this end it is very convenient to choose a basis for the six constant two-forms comprising three self-dual ω^a and as many anti self-dual $\bar{\omega}^a$ two-forms and decompose F according to

$$F = \frac{1}{2} F_{ij} dx^i dx^j = F^+ + F^- = f_r \omega^r + \bar{f}_r \bar{\omega}^r \,. \tag{4.1}$$

This can be rather explicitly achieved, for instance, by using 't Hooft symbols η^a_{mn} and $\bar{\eta}^r_{ij}$, with $r = 1,2,3$ and $i,j = 1,\dots 4$, and setting $\omega^r = \eta^r_{ij} dx^i dx^j/2$ and $\bar{\omega}^r = \bar{\eta}^r_{ij} dx^i dx^j/2$. 't Hooft symbols are real and antisymmetric and satisfy

$$\begin{aligned} \eta_i^{rk} \eta_k^{sj} &= -\delta^{rs} \delta_i{}^j + \epsilon^{rs}{}_t \eta_i^{tj} \\ \bar{\eta}_i^{rk} \bar{\eta}_k^{sj} &= -\delta^{rs} \delta_i{}^j + \epsilon^{rs}{}_t \bar{\eta}_i^{tj} \\ \eta_i^{rk} \bar{\eta}_k^{sj} &= \bar{\eta}_i^{sk} \eta_k^{rj} \,. \end{aligned} \tag{4.2}$$

In terms of Pauli matrices it is easy to check that

$$\begin{array}{lll} \eta^1 = i\sigma_2 \otimes \sigma_1 & \eta^2 = i\sigma_2 \otimes \sigma_3 & \eta^3 = 1 \otimes i\sigma_2 \\ \bar{\eta}^1 = \sigma_1 \otimes i\sigma_2 & \bar{\eta}^2 = \sigma_3 \otimes i\sigma_2 & \bar{\eta}^3 = i\sigma_2 \otimes 1 \end{array} \tag{4.3}$$

do the job.

For singly charged strings, the magnetic rotation matrices, whose eigenvalues determine the magnetic shifts, assume the form

$$R = (1-H)(1+H)^{-1} = (1 - H^+ - H^-)(1 + H^+ + H^-)^{-1} = R_L R_R = R_R R_L \tag{4.4}$$

where $H_{\hat{i}\hat{j}} = E^i_{\hat{i}} E^j_{\hat{j}} F_{ij}$ are frame components. After straightforward algebra, that heavily relies on the use of $[H^+, H^-] = 0$, $(H^+)^2 = -|h|^2 \mathbb{1}$ and $(H^-)^2 = -|\bar{h}|^2 \mathbb{1}$, with $|v|^2 \equiv \sum_r v_r^2$, one finds

$$R_L = \frac{(1 - |h|^2 + |\bar{h}|^2) - 2H^+}{\sqrt{1 + 2(|h|^2 + |\bar{h}|^2) + (|h|^2 - |\bar{h}|^2)^2}} \tag{4.5}$$

$$R_R = \frac{(1 + |h|^2 - |\bar{h}|^2) - 2H^-}{\sqrt{1 + 2(|h|^2 + |\bar{h}|^2) + (|h|^2 - |\bar{h}|^2)^2}} \,. \tag{4.6}$$

Here the denominator reproduces the Born-Infeld action.

The four bosonic string coordinates transform in the $(1/2, 1/2)$ representation of SO(4) $=$ SU(2)$_L \times$ SU(2)$_R$, and thus the magnetic shifts of the modes of singly charged spacetime bosons, with one end on an unmagnetized stack, are determined by the two pairs

$$\beta_{Q=1} = \pm(\beta_L \pm \beta_R) \tag{4.7}$$

with

$$\tan(2\beta_L) = \frac{2|h|}{1 - |h|^2 + |\bar{h}|^2} \,, \qquad \tan(2\beta_R) = \frac{2|\bar{h}|}{1 + |h|^2 - |\bar{h}|^2} \,. \tag{4.8}$$

Spacetime spinors transforming in the $(1/2, 0)$, respectively $(0, 1/2)$, representation have their masses shifted by $\pm\beta_L/\pi$, respectively $\pm\beta_R/\pi$. Thus the only way to preserve some susy in this (singly charged) sector is having either $\beta_L = 0$ or $\beta_R = 0$, which is tantamount to turning on only (anti) self-dual magnetic fields, i.e. abelian (anti)instantons on $\mathbf{T}^4$.

In the self-dual case, choosing as reference complex structure, e.g. $J_{ij} = \bar{\eta}^3_{ij}$, i.e. setting $z^1 = x^1 + ix^2$ and $z^2 = x^3 + ix^4$, shows that the magnetic two-form is a type $(1,1)$ form, i.e. $F_{(2,0)} = 0$, with $F_{(1,1)} \wedge J = 0$. In mathematical terms, one is dealing with a hermitean connection on a holomorphic stable line bundle $\mathcal{E}$. The choice of complex coordinates, though particularly suitable to exposing susy and reducing the problem to a two-dimensional one, hides the beautiful hyperkähler structure coded in the ADHM construction of instantons on $\mathbf{T}^4$ or on its susy preserving orbifolds $\mathbf{T}^4/\Gamma \approx \mathbf{K}3$.

For doubly charged strings, the situation is analogous and one gets

$$\beta_{Q=2} = \pm 2(\beta_L \pm \beta_R) \tag{4.9}$$

for spacetime bosons and $\beta_{Q=2} = \pm 2\beta_L$ or $\beta_{Q=2} = \pm 2\beta_R$ for spacetime fermions.

The situation is slightly more involved for open strings ending on branes with different magnetic fields, i.e. carrying charge $q_a = \pm 1$ with respect to the $\mathrm{U}(1)_a$ subgroup associated to the first stack of N_a branes and charge $q_b = \pm 1$ with respect to the $\mathrm{U}(1)_b$ subgroup associated to the second stack of N_b branes. Yet exploiting the close analogy between 't Hooft symbols, or rather the 4×4 matrices $\Sigma^a = i\eta^a$ and $\bar{\Sigma}^a = i\bar{\eta}^a$, and Pauli matrices σ^a, it takes a few minutes thinking to convince oneself that the relevant matrices

$$R_{abL} = R^{q_a}_{aL} R^{q_b}_{bL}\,, \qquad R_{abR} = R^{q_a}_{aR} R^{q_b}_{bR} \tag{4.10}$$

that determine $R_{ab} = R_{abL} R_{abR} = R_{abR} R_{abL}$ can be conveniently expressed in the form

$$R^q_L = \cos(2\beta_L) - iq\sin(2\beta_L)\hat{h}\cdot\Sigma\,, \qquad R^q_R = \cos(2\beta_R) - iq\sin(2\beta_R)\hat{\bar{h}}\cdot\bar{\Sigma} \tag{4.11}$$

where $\tan(2\beta_{L/R})$ have been defined above and a hat over a three-component vector means the unit vector in its direction. Denoting for short $\cos(2\beta_{a,b})$, respectively $\sin(2\beta_{a,b})$, by $c_{a,b}$, respectively $s_{a,b}$, and combining the two rotations yields

$$R_{abL} = (c_a c_b - q_a q_b s_a s_b \hat{h}_a \cdot \hat{h}_b) - i(q_a s_a c_b \hat{h}_a + q_b c_a s_b \hat{h}_b + q_a q_b s_a s_b \hat{h}_a \times \hat{h}_b)\cdot\Sigma \tag{4.12}$$

and

$$R_{abR} = (\bar{c}_a \bar{c}_b - q_a q_b \bar{s}_a \bar{s}_b \hat{\bar{h}}_a \cdot \hat{\bar{h}}_b) - i(q_a \bar{s}_a \bar{c}_b \hat{\bar{h}}_a + q_b \bar{c}_a \bar{s}_b \hat{\bar{h}}_b + q_a q_b \bar{s}_a \bar{s}_b \hat{\bar{h}}_a \times \hat{\bar{h}}_b)\cdot\bar{\Sigma} \tag{4.13}$$

The magnetic shifts of the modes of the four bosonic coordinates (two complex pairs)

$$\beta_{ab} = \pm\beta_{abL} \pm \beta_{abR} \tag{4.14}$$

are determined by

$$\cos(2\beta_{abL}) = c_a c_b - q_a q_b s_a s_b \hat{h}_a \cdot \hat{h}_b\,, \qquad \cos(2\beta_{abR}) = \bar{c}_a \bar{c}_b - q_a q_b \bar{s}_a \bar{s}_b \hat{\bar{h}}_a \cdot \hat{\bar{h}}_b\,. \tag{4.15}$$

Generically, there are no unmagnetized directions. Indeed $\beta_{ab} \neq 0$ unless $h_a = \pm\bar{h}_a$, in which case one is effectively restricting the fluxes to a three-dimensional torus and considering the diagonal SO(3) subgroup. Strings connecting branes of different kinds may however appear with non-trivial multiplicities associated to the presence of fermionic zero-modes counted by the index of the Dirac operator coupled with charge q_a to the magnetic field F_a on the first stack of N_a branes and with charge q_b to the magnetic field F_b on the second stack of N_b branes. The index vanishes in odd dimension. In four dimensions it is given by the second Chern number $c_2(\mathcal{E}_{ab})$ of the abelian gauge bundle $\mathcal{E}_{ab} = \mathcal{E}_a^{q_a} \otimes \mathcal{E}_b^{q_b}$

$$\begin{aligned} I_{ab} &= -\frac{1}{2(2\pi)^2}\int_{\Sigma^4}(q_a\mathcal{F}_a + q_b\mathcal{F}_b)\wedge(q_a\mathcal{F}_a + q_b\mathcal{F}_b) \\ &= \frac{W_aW_b}{(2\pi)^2}[-|q_ah_a + q_bh_b|^2 + |q_a\bar{h}_a + q_b\bar{h}_b|^2]Vol(\Sigma^4)\,, \end{aligned} \tag{4.16}$$

where $\mathcal{F}_{a/b}$ are the world-volume magnetic fields and $W_{a/b} = \det(W_{a/b})$ take care of the oriented brane wrappings so that $N_{a/b} \to N_{a/b}|W_{a/b}|$. For "parallel" magnetic fields in factorized rectangular two-tori $\mathbf{T}^4 = \mathbf{T}^2_{(1)} \times \mathbf{T}^2_{(2)}$, $W_{a/b} = \prod_{I=1}^2 n^{(I)}_{a/b}$ and (4.16) reduces to the simpler expression

$$I_{ab} = \frac{W_aW_b}{(2\pi)^2}\prod_{I=1}^{2}\left(q_ah_a^{(I)} + q_bh_b^{(I)}\right)Vol\left(\mathbf{T}^2_I\right) = \prod_{I=1}^{2}\left(q_am_a^{(I)}n_b^{(I)} + q_bm_b^{(I)}n_a^{(I)}\right) \tag{4.17}$$

upon imposing Dirac quantization condition on each subtorus and for each stack of branes. In summary, there are I_{ab} strings in the representation $(N_a, N_b{}^*)$ or (N_a, N_b), present only in the unoriented case, of the Chan-Paton group commuting with the magnetic U(1)'s. The analysis of the doubly charged strings, present only in the unoriented case, is similar. In order to correctly determine the multiplicities one has to properly take into account the action of Ω that (anti) symmetrizes the Chan-Paton indices.

Although we have all the necessary ingredients to write down the one-loop partition function for a magnetized four-torus, we refrain from doing so in order to avoid cumbersome formulae that can be found in [30] and conveniently subsumed by the following observations. As in the two-dimensional case, one has to distinguish several sectors.

Neutral strings with $q_a = q_b = 0$, starting and ending on unmagnetized branes, suffer neither magnetic shifts of their oscillator modes nor rescalings of their KK momenta.

Dipole strings with $q_a = -q_b = \pm 1$, starting and ending on the same stack of magnetized branes, suffer no magnetic shifts but carry rescaled KK momenta.

Singly charged strings $q_a = \pm 1$ and $q_b = 0$ or *vice versa*, starting on a stack of magnetized branes and ending on a stack of unmagnetized branes, suffer magnetic shifts given by (4.8) and, generically, carry no KK momenta but rather multiplicities determined by the index theorem (4.16) to be $I_{ao} = c_2(\mathcal{E}_a^{q_a})$.

Doubly charged strings with $q_a = q_{\tilde{a}} = \pm 1$, starting on magnetized branes and ending on their images under Ω, suffer double magnetic shifts and, generically, carry no KK momenta but rather multiplicities determined by the index theorem to be $I_{aa} = c_2(\mathcal{E}_a^{2q_a}) = 4c_2(\mathcal{E}_a^{q_a})$.

Dy-charged strings with $q_a = \pm 1$ and $q_b = \pm 1$, starting on a stack of magnetized branes and ending on a different stack of magnetized branes or their images under Ω, suffer magnetic shifts determined by the trigonometric formula (4.15), generically carry no KK momenta but rather discrete multiplicities determined by the index theorem to be $I_{ab} = c_2(\mathcal{E}_a^{q_a} \otimes \mathcal{E}_b^{q_b})$.

So far we have tacitly assumed single wrapping, i.e. $X^i = \delta^i_\alpha \sigma^\alpha$, for all the branes. In the case where $W^i_{(a)\alpha} \neq \delta^i_\alpha$ for some a but still in the absence of magnetic field, the integers $W_a = \det({W_\alpha}^i_{(a)})$ simply rescale the tension of the stack of branes $\tau_a = \tau N_a |W_a|$ and their R-R charge density $\rho_a = \tau N_a W_a$. The stacks thus preserve the same 16 supercharges only when $W_a W_b > 0$ for all a and b. Moreover in type I theories this fraction coincides with the one preserved by standard Ω-planes for a given choice of sign, let us say $W_a > 0$. For vanishing NS-NS antisymmetric tensor background, R-R tadpole cancellation requires $2\sum_a N_a W_a = 32$, after including image branes. The situation becomes more interesting when some magnetic fields are turned on. The subtle point is that the magnetic field that satisfies Dirac quantization condition is the worldvolume magnetic field $\mathcal{F}^a_{\alpha\beta}$ rather than F_{ij}, that need not have integer 'periods', i.e. first Chern numbers c_{ij}. Rather $c_{\alpha\beta} = {W_\alpha}^i {W_\beta}^j c_{ij}$ are integrally quantized, except possibly for a half-integer shift related to the presence of (the pull-back of) a quantized NS-NS antisymmetric tensor background $\mathcal{B}_{\alpha\beta} = {W_\alpha}^i {W_\beta}^j B_{ij}$. As observed by [24], this may not be sufficient. In general, one has to require integrality of all Chern numbers, which are T-dual to all possible (sub)brane charges. In practise, one has to solve the inverse problem, i.e. given rational numbers c^1_{ij}, $c^2_{ij,kl}$, etc, one should find an integer matrix ${W_\alpha}^i$ such that $c^1_{\alpha\beta}$, $c^2_{\alpha\beta,\gamma\delta}$, etc are (half) integer compatibly with R-R tadpole conditions. This may fail with purely abelian fields. In some cases [24], adding a non-abelian part might help imposing integrality but would require abandoning the exact worldsheet analysis of the present paper.

We are now ready to discuss the potential for the closed string moduli and address the issue of their stabilization at the origin of the open string moduli space. Analyzing the mixing with open string states is beyond the scope of the present investigation. The potential generated by the abelian Born-Infeld lagrangian is given by [40]

$$\sqrt{\det(\mathcal{G}_{\alpha\beta} + \mathcal{F}_{\alpha\beta})} = \sqrt{\det[{W_\alpha}^i {W_\beta}^j (G_{ij} + F_{ij})]} = |\det(W^i_\alpha)| \sqrt{\det(G_{ij} + F_{ij})} \tag{4.18}$$

and the WZ coupling to the R-R fields reads

$$\begin{aligned}\int_{\Sigma^d} \sum_k \mathcal{C}_{k+1} \operatorname{Tr}(e^{\mathcal{F}}) = \sum_a W_a \Big[C_{10-d} \int_{\mathbf{T}^d} \omega_d N_a + \sum_f C^f_{8-d+2} \int_{\mathbf{T}^d} \chi^r_{d-2} \wedge F_a + \\ + \sum_p C^q_{6-d+2} \int_{\mathbf{T}^d} \psi^q \wedge F_a \wedge F_a + \\ + \sum_u C^u_{4-d+2} \int_{\mathbf{T}^d} \varphi^u \wedge F_a \wedge F_a \wedge F_a \Big]\end{aligned} \tag{4.19}$$

Terms with an odd number of magnetic fields disappear upon Ω-projection. Notice that once $\det(W^i_\alpha)$ is factored out it is the non-integrally (but rationally) quantized F_a that appears. The Born-Infeld term [40] yields

$$\sqrt{\det(\mathcal{G}_{ij}+\mathcal{F}_{ij})} = \sqrt{\det(\mathcal{G}_{ij})}\sqrt{\det(\mathbb{1}+\mathcal{H})} \tag{4.20}$$

where $\mathcal{H}_{\hat{i}\hat{j}} = \mathcal{E}^i_{\hat{i}}\mathcal{E}^j_{\hat{j}}\mathcal{F}_{ij}$. In turn

$$\det(\mathbb{1}+\mathcal{H}) = \exp\left[\mathrm{tr}(\log(\mathbb{1}+\mathcal{H})\right] = \exp\left[\sum_{k=0}^{[d/2]}\frac{1}{2k}\,\mathrm{tr}(\mathcal{H}^{2k})\right] \tag{4.21}$$

where tr(..) denote a trace over target space vector indices, not to be confused with Tr(..) denoting a trace over Chan-Paton indices. Indeed it is easy to show that $\mathrm{tr}(\mathcal{H}^{2k+1}) = 0$. Similarly the restriction to $k < [d/2]$ can be reached via group theory considerations as well as inspection of the initial determinant that can expose terms with at most $\mathcal{F}^d$.

More explicitly, for $d = 2, 3$

$$\frac{\det(\mathcal{G}+\mathcal{F})}{\det(\mathcal{G})} = \left[1-\frac{1}{2}\,\mathrm{tr}(\mathcal{H}^2)\right] = (1+h^2) \tag{4.22}$$

where h is the skew eigenvalue of $\mathcal{H}$.

Similarly, for $d = 4, 5$, one has

$$\frac{\det(\mathcal{G}+\mathcal{F})}{\det(\mathcal{G})} = \left[1-\frac{1}{2}\,\mathrm{tr}(\mathcal{H}^2)-\frac{1}{4}\,\mathrm{tr}(\mathcal{H}^4)+\frac{1}{8}(\mathrm{tr}(\mathcal{H}^2))^2\right] = (1+h_1^2)(1+h_2^2) \tag{4.23}$$

where h_I with $I = 1, 2$ are the two skew eigenvalues of $\mathcal{H}$.

Finally, for $d = 6, 7$, one finds

$$\begin{aligned}\frac{\det(\mathcal{G}+\mathcal{F})}{\det(\mathcal{G})} &= \left[1-\frac{1}{2}\,\mathrm{tr}(\mathcal{H}^2)-\frac{1}{4}\,\mathrm{tr}(\mathcal{H}^4)+\frac{1}{8}(\mathrm{tr}(\mathcal{H}^2))^2-\right.\\ &\left.\quad-\frac{1}{6}\,\mathrm{tr}(\mathcal{H}^6)+\frac{1}{8}\,\mathrm{tr}(\mathcal{H}^2)\,\mathrm{tr}(\mathcal{H}^4)-\frac{1}{48}(\mathrm{tr}(\mathcal{H}^2))^3\right] = \prod_{I=1}^{3}(1+h_I^2)\end{aligned} \tag{4.24}$$

where h_I with $I = 1, 2, 3$ are the skew eigenvalues of $\mathcal{H}$.

Let us now specialize to $d = 4$, where one can exploit the hyperkähler structure of $\mathbf{T}^4$ and decompose $\mathcal{H}$ into its self-dual and anti-self-dual parts $\mathcal{H} = \mathcal{H}_+ + \mathcal{H}_-$. Then one can rewrite (4.23) as

$$1-\frac{1}{2}\,\mathrm{tr}\left(\mathcal{H}^2\right)-\frac{1}{4}\,\mathrm{tr}\left(\mathcal{H}^4\right)+\frac{1}{8}(\mathrm{tr}(\mathcal{H}^2))^2 = 1-\frac{1}{2}\left[\mathrm{tr}\left(\mathcal{H}_+^2\right)+\mathrm{tr}\left(\mathcal{H}_-^2\right)\right]+\frac{1}{16}\left[\mathrm{tr}(\mathcal{H}_+^2)-\mathrm{tr}\left(\mathcal{H}_-^2\right)\right]^2. \tag{4.25}$$

Combining with the volume factor, the Born-Infeld lagrangian of each stack of branes can be elegantly written in the form

$$\begin{aligned}&\sqrt{\det(\mathcal{G})}\left[\left(1\mp\frac{1}{2}[\mathrm{tr}(\mathcal{H}_+^2)\right.-\mathrm{tr}(\mathcal{H}_-^2)]\right)^2+\mathrm{tr}(\mathcal{H}_\mp^2)\right]^{1/2} =\\ &= |W|\left[\left(\frac{1}{2}\int J\wedge J-\frac{1}{2}\int F\wedge F\right)^2+\left(\int J\wedge F\right)^2+\left|\int\Omega\wedge F\right|^2\right]^{1/2}\end{aligned} \tag{4.26}$$

where $\Omega = J^1 + iJ^2 = J^+$ is the complex (2,0)-form and $J = J^3$ is the Kähler form in the reference complex structure. Our normalizations are such that $\Omega \wedge \bar{\Omega} = 2J \wedge J$. At the linear level the susy preserving BPS conditions require that either $\mathcal{H}_+ = 0$ or $\mathcal{H}_- = 0$ for all magnetic U(1)'s. In the latter case, $\mathcal{H}_- = 0$, the last three terms drop out and one ends up with

$$\sqrt{\det(\mathcal{G})}\left|1 - \frac{1}{2}\operatorname{tr}(\mathcal{H}_+^2)\right| = \sqrt{\det(\mathcal{G})}\left(1 - \frac{1}{2}\operatorname{tr}(\mathcal{H}_+^2)\right).$$

Indeed, both terms are positive definite since $\mathcal{H}_+$ being real and antisymmetric is anti-hermitean. The same is true in the former case, $\mathcal{H}_+ = 0$, after reversing the orientation of $\mathbf{T}^4$.

Contrary to the 2- and 3-dimensional cases, the 1/2-loop potential for the NS-NS moduli induced by the Born-Infeld term combined with the contribution of Ω-planes depends *a priori* on all the moduli. Rescaling the metric according to $\widehat{G} = G/\det(G)^{1/4}$, it can be cast into the form

$$V(\phi, \omega, \hat{G}; \mathcal{F}_a) = \tau e^{-\phi}\left(2\sum_a N_a \sqrt{|W_a|^2\omega^2 + |W_a|\omega c_1^2(\mathcal{E}_a) + c_2^2(\mathcal{E}_a)} \mp 32|\omega|\right) \tag{4.27}$$

where $\omega = \sqrt{\det(G)}$ parametrizes the overall volume of $\mathbf{T}^4$. Depending on whether $c_1^2(\mathcal{E}_a)$ is larger or smaller than $2|c_2(\mathcal{F}_a)|$ and on the sign of the Ω-plane tension one gets different behaviours for ω. For supersymmetric configurations $c_1^2(\mathcal{E}_a) = \pm 2c_2(\mathcal{E}_a)$ (the same sign for all a) and one simply gets

$$V(\phi, \omega, \hat{G}; F_a) = \tau e^{-\phi}\left(2\sum_a N_a(|W_a||\omega| + |c_2(\mathcal{E}_a)|) \mp 32|\omega|\right) \tag{4.28}$$

this should be supplemented with the R-R tadpole conditions $2\sum_a N_a W_a = \pm 32$ (for C_{10}) and $2\sum_a N_a W_a c_2(E_a) = 0$ (for C_6) that cannot be satisfied for non vanishing magnetic fields unless one introduces lower dimensional Ω-planes.

For non supersymmetric configurations, R-R tadpoles can be satisfied and in principle all the closed string moduli, including ω yet excluding ϕ, can be stabilized in the presence of enough stacks of branes with *oblique* magnetic fluxes. One cannot prevent however the presence of open string tachyons that destabilize the vacuum anyway.

We would like to conclude this section with an amusing observation on non-linear self-duality [23 − 25]. As discussed above, barring the overall volume factor, the Born-Infeld action in $d = 4$ can be compactly written as

$$\sqrt{\det(1 + \mathcal{H})} = \sqrt{(1 + h_1^2)(1 + h_2^2)} = \sqrt{(1 \pm h_1 h_2)^2 + (h_1 \mp h_2)^2} \tag{4.29}$$

where h_I with $I = 1, 2$ are the two skew eigenvalues of H. "Standard" (anti) self-dual and thus supersymmetric configurations correspond to $h_1 \mp h_2 = 0$ and yield

$$\sqrt{\det(1 + \mathcal{H})} = |1 \pm h_1 h_2| = 1 + |h|^2 \tag{4.30}$$

that, combined with the R-R coupling, disclose the presence of BPS D5- or $\bar{\text{D}}$5-branes inside the magnetic D9-branes [22]. The resulting 1/4 BPS bound state is at threshold since the positive tensions simply add. In order to 'dispose of' the square root in (4.29) and find other bound states at threshold one can however consider the case $1 \pm h_1 h_2 = 0$, i.e. $h_1 = \pm 1/h_2$ that does not lead to standard BPS branes but corresponds to a non (manifestly) BPS state which is T-dual to a BPS configuration of D5-branes bound to pairs of D9- and $\bar{\text{D}}$9-branes. It is natural to ask to what extent the initial configuration satisfying the non-linear (anti) self-duality condition is non-BPS. It is not unconceivable that BPS-ness can be regained, even before perfoming any T-duality, with respect to local supersymmetries associated to gravitini in the closed string spectrum that become massless at the non-linear (anti) self-dual point.

5. Five-dimensional magnetized torus

A constant magnetic field H in five dimensions has 10 independent components. Four of them may be taken to be associated to the independent components of the unit vector $\vec{u}$ along the "unmagnetized" direction associated to the unit eigenvalue of the rotation matrix $R(H)$. The other six determine the magnetic field in the four transverse generically "magnetized" directions. By means of this splitting one can easily determine the magnetic shifts for singly and doubly charged strings and the rescaling of the momenta for dipole strings. The study of dy-charged strings, although straightforward in principle, is quite laborious in practice since one has to diagonalize a generic 5×5 orthogonal matrix, something that leads to very cumbersome formulae. Yet, the strategy is first to identify the "unmagnetized" direction by solving $R_{ab}\vec{u}_{ab} = \vec{u}_{ab}$, then decompose the reduced 4×4 orthogonal matrix according to $R^{\perp}_{ab} = R^{\perp L}_{ab} R^{\perp R}_{ab}$ and finally exploit the properties of 't Hooft symbols in order to determine the eigenvalues and the resulting magnetic shifts.

The condition for supersymmetry in $D = 5$ are as stringent as in $D = 6$. In concrete terms the magnetic rotation matrices should all belong to the same $\mathrm{SU}(2) = Sp(2)$ subgroup of $\mathrm{SO}(5) = Sp(4)$. In particular this means that they should all have the same unmagnetized direction $\vec{u}$, which may or may not coincide with one of the generators of the fundamental cell. The holonomy of the SO(4) stability group should further reduce to SU(2).

Since nothing essentially new can be learned from this case we do not go into any further detail.

6. Six-dimensional magnetized torus

Six dimensional magnetized tori represent a genuinely new and phenomenologically interesting case. In general, a magnetic field has 15 independent components, corresponding to the constant and thus closed two-forms. For obvious (supersymmetry) purposes it is convenient to choose complex coordinates and set

$$F^a = F^a_{(2,0)} + F^a_{(1,1)} + F^a_{(0,2)} \tag{6.1}$$

where $F^a_{(2,0)}$, $F^a_{(1,1)}$ and $F^a_{(0,2)}$ have 3, 9 and 3 independent components respectively. Notice that $F^a_{(0,2)} = F^{a*}_{(2,0)}$, $F^a_{(1,1)} = F^{a*}_{(1,1)}$ for a real gauge field, i.e. a hermitean connection.

In order to preserve some supersymmetry [23, 21, 25], the complex structure of $\mathbf{T}^6$ must be chosen so that for all a

$$F^a_{(0,2)} = 0 \tag{6.2}$$

and the Kähler form $J = iG_{I\bar{J}} dz^I \wedge dz^{\bar{J}}$ must satisfy

$$\frac{1}{2!} J \wedge J \wedge F^a_{(1,1)} = \frac{1}{3!} F^a_{(1,1)} \wedge F^a_{(1,1)} \wedge F^a_{(1,1)} \,, \tag{6.3}$$

i.e. the magnetic bundle $\mathcal{E}$ must be holomorphic and stable wrt to J. In turn this implies a reduction of the holonomy group generated by the magnetic rotation matrices R_a from SO(6) = SU(4) to SU(3) for $\mathcal{N} = 1$ supersymmetry or SU(2) for $\mathcal{N} = 2$ supersymmetry. $\mathcal{N} = 4$ supersymmetry is enjoyed by the neutral open strings, for which $R_0 = \mathbb{1}$.

In the supersymmetric case, the 6×6 rotation matrices are direct sums of 3×3 blocks

$$R = \begin{pmatrix} U_{\mathbf{3}} & 0 \\ 0 & U_{\mathbf{3}^*} \end{pmatrix} \tag{6.4}$$

since $\mathbf{6} \to \mathbf{3} + \mathbf{3}^*$ under SU(4) $\to$ U(3). More explicitly

$$U_{\mathbf{3}}{}^I{}_J = (\delta^I{}_K - iH^I{}_K)(\delta^K{}_J + iH^K{}_J)^{-1} \tag{6.5}$$

where raising and lowering of frame indices is trivially accomplished by means of $\delta^{I\bar{J}}$ and $\delta_{I\bar{J}}$.

The stability condition is not simply $H^I{}_I = 0$, as in $d = 4$ where it arose from $J \wedge F = 0$, but rather the condition for SU(3) holonomy: $\det(U_{\mathbf{3}}) = 1$. In other words $\det(1 + iH) = \det(1 - iH)$ that implies$\sum_k (-)^k \operatorname{tr}(H^{2k+1})/(2k+1) = 0$. Since in general $\operatorname{tr}_{\mathbf{3}}(H) \neq 0$, one can expand $H_{\mathbf{3}}$ in the basis of generators of U(3)

$$H = h_0 \mathbb{1} + h_r \lambda^r \tag{6.6}$$

where we choose the eight 3×3 Gell-Mann matrices λ^r so as to be hermitean and satisfy

$$\operatorname{tr}(\lambda^r \lambda^s) = 3\delta^{rs} \qquad \lambda^r \lambda^s = \delta^{rs} \mathbb{1} + (if^{rst} + d^{rst})\lambda_t \,, \tag{6.7}$$

where the SU(3) structure constants f^{rst} are totally antisymmetric and the anomaly coefficients d^{rst} are totally symmetric.

In order to determine the eigenvalues $\rho_I = \exp(2i\beta_I)$ of U in terms of the 9 components h_0 and h_r of H it is convenient to diagonalize $\hat{H} = h_r \lambda^r = h \cdot \lambda$ so that

$$\rho_I = \frac{1 - ih_0 - i\mu_I}{1 + ih_0 + i\mu_I} \tag{6.8}$$

Using $\operatorname{tr}(\hat{H}) = 0$, $\operatorname{tr}(\hat{H}^2) = 3\delta^{rs} h_r h_s \equiv 3|h|^2$ and $\operatorname{tr}(\hat{H}^3) = 3d^{rst} h_r h_s h_t \equiv 3dhhh$, the secular equation for $\hat{H}$ boils down to a cubic in canonical form

$$\mu^3 - \frac{3}{2}|h|^2 \mu - dhhh = 0 \,, \tag{6.9}$$

whose solutions are given by Cardano - Tartaglia formula

$$\mu_I = \frac{e^{2I\pi i/3}}{2^{1/3}} \left[dhhh + i\sqrt{\frac{1}{2}|h|^6 - (dhhh)^2} \right]^{1/3} + \frac{e^{-2I\pi i/3}}{2^{1/3}} \left[dhhh - i\sqrt{\frac{1}{2}|h|^6 - (dhhh)^2} \right]^{1/3} , \tag{6.10}$$

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where $|h|^6 \geq 2(dhhh)^2$ has been used. Supersymmetry translates into the condition

$$h_0^3 - 3h_0(|h|^2/2 + 1) + dhhh = 0\,. \tag{6.11}$$

We now consider the more laborious case of strings ending on different stacks of branes, whose magnetic shifts are determined by the eigenvalues of the SU(3) matrix $U_{ab} = U_a^{q_a} U_b^{q_b}$. Setting for definiteness $q_a = 1$ and $q_b = -1$ and factoring out an overall U(1) phase gives

$$\begin{aligned} U_{ab} &= \frac{(1 - ih_a^0)(1 + ih_b^0)}{(1 + ih_a^0)(1 - ih_b^0)}(1 - i\hat{h}_a \cdot \lambda)(1 + i\hat{h}_a^* \cdot \lambda)^{-1}(1 - i\hat{h}_b^* \cdot \lambda)(1 + i\hat{h}_b \cdot \lambda)^{-1} \\ &= \frac{(1 - ih_a^0)(1 + ih_b^0)}{(1 + ih_a^0)(1 - ih_b^0)}\,\hat{U}_{ab} \end{aligned} \tag{6.12}$$

where $\hat{h}_{a/b} = h_{a/b}/(1 - ih_{a/b}^0)$ and $*$ denotes complex conjugation. Perusing (6.7) yields

$$\hat{U}_{ab} = M_a^{-1}\frac{1 - \hat{h}_a \cdot \hat{h}_b}{1 - \hat{h}_a^* \cdot \hat{h}_b^*}(1 - i\hat{h}_{ab} \cdot \lambda)(1 + i\hat{h}_{ab}^* \cdot \lambda)^{-1}M_a \tag{6.13}$$

where $M_a = 1 + i\hat{h}_a^* \cdot \lambda$ and

$$\hat{h}_{ab} = \frac{\hat{h}_a + \hat{h}_b + f\hat{h}_a\hat{h}_b - id\hat{h}_a\hat{h}_b}{1 - \hat{h}_a \cdot \hat{h}_b} \tag{6.14}$$

so that

$$\hat{\mu}_{ab}^I = \frac{1 - \hat{h}_a \cdot \hat{h}_b}{1 - \hat{h}_a^* \cdot \hat{h}_b^*}\hat{\nu}_{ab}^I \tag{6.15}$$

where

$$\begin{aligned} \hat{\nu}_{ab}^I = & \frac{e^{2I\pi i/3}}{2^{1/3}}\left[d\hat{h}_{ab}\hat{h}_{ab}\hat{h}_{ab} + i\sqrt{\frac{1}{2}|\hat{h}_{ab}|^6 - (d\hat{h}_{ab}\hat{h}_{ab}\hat{h}_{ab})^2}\right]^{1/3} + \\ & + \frac{e^{-2I\pi i/3}}{2^{1/3}}\left[d\hat{h}_{ab}\hat{h}_{ab}\hat{h}_{ab} - i\sqrt{\frac{1}{2}|\hat{h}_{ab}|^6 - (d\hat{h}_{ab}\hat{h}_{ab}\hat{h}_{ab})^2}\right]^{1/3} \end{aligned} \tag{6.16}$$

and eventually one can compute $\rho_{ab}^I = \exp(2i\beta_{ab}^I)$ and get the magnetic shifts $\epsilon_{ab}^I = \beta_{ab}^I/\pi$, where $I = 1, 2, 3$ labels the three orthogonal directions along the eigenevectors of U_{ab}. Clearly the eigenvalues of R come in complex conjugate pairs $(\rho_I, \bar{\rho}_I)$. Though slightly unfamiliar, the explicit and to some extent compact expressions for the eigenvalues of H and R may prove very useful in extracting the spectrum and some couplings. As an illustration of the procedure, we have computed all the magnetic shifts relevant to determine the spectrum of the $\mathcal{N} = 1$ supersymmetric AM model constructed by Antoniadis and Maillard [21]. The not so inspiring results are gathered in the appendix A.

As usual multiplicities, associated to the degeneracy of the Landau levels, are counted by the relevant index theorem. In six dimensions it is given by the third Chern number $c_3(\mathcal{E}_{ab})$ of the abelian gauge bundle $\mathcal{E}_{ab} = \mathcal{E}_a^{q_a} \otimes \mathcal{E}_b^{q_b}$

$$\begin{aligned} I_{ab} &= -\frac{1}{3!(2\pi)^3}\int_{\Sigma^6}(q_a\mathcal{F}_a + q_b\mathcal{F}_b) \wedge (q_a\mathcal{F}_a + q_b\mathcal{F}_b) \wedge (q_a\mathcal{F}_a + q_b\mathcal{F}_b) \\ &= -\frac{W_aW_b}{3!(2\pi)^3}\int_{\mathbf{T}^6}(q_aF_a + q_bF_b) \wedge (q_aF_a + q_bF_b) \wedge (q_aF_a + q_bF_b)\,. \end{aligned} \tag{6.17}$$

For "parallel" magnetic fields in factorized rectangular two-tori $\mathbf{T}^6 = \prod_{I=1}^3 \mathbf{T}^2_{(I)}$, $W_{a/b} = \prod_{I=1}^3 n^{(I)}_{a/b}$ and (6.17) reduces to the simpler expression

$$I_{ab} = \frac{W_a W_b}{(2\pi)^3} \prod_{I=1}^{3} \left(q_a h_a^{(I)} + q_b h_b^{(I)} \right) Vol(\mathbf{T}^2_I) = \prod_{I=1}^{3} \left(q_a m_a^{(I)} n_b^{(I)} + q_b m_b^{(I)} n_a^{(I)} \right) \tag{6.18}$$

upon imposing Dirac quantization condition on each subtorus and for each stack of branes. When $I_{ab} = 0$ one has no chiral asymmetry that may sometimes signal the presence of one or more unmagnetized directions. This is the case e.g. for the AM model described in the appendix. One can define a reduced index $I^{\perp}_{ab}$ in the subspace orthogonal to the fixed tori that count their number. Clearly in such cases, dycharged strings can carry generalized KK momenta in the unmagnetized directions.

We then turn to address the issue of moduli stabilization that relies on the potential for the closed string moduli generated by the abelian and thus mutually commuting (in the Chan-Paton sense!) magnetic fluxes. In order to avoid facing subtleties with the non-abelian extension of the non-abelian Born-Infeld action, we do not turn on VEV's for any, charged or neutral, open string massless state. The abelian Born-Infeld action, compactly expressed in terms of the skew eigenvalues of $H_{\mathbf{6}}$ as in (4.24), can be rewritten as

$$\sqrt{\prod_I (1 + h_I^2)} = \sqrt{(1 - h_1 h_2 - h_2 h_3 - h_3 h_1)^2 + (h_1 h_2 h_3 - h_1 - h_2 - h_3)^2} \tag{6.19}$$

up to an overall factor of $\sqrt{\det(\mathcal{G})}$. Expressing h_I in terms of the available SO(6) invariants, i.e. $\mathrm{tr}(H^2)$, $\mathrm{tr}(H^4)$, and $\mathrm{tr}(H^6)$ or, equivalently, $\varepsilon HHH = -\,\mathrm{tr}(H^6)/6 + \mathrm{tr}(H^2)\,\mathrm{tr}(H^4)/8 - \mathrm{tr}(H^2)^3/48$, requires solving a complete cubic equation. For $F_{(2,0)} = 0$, we have already solved the problem, since $H_{\mathbf{6}} = H_{\mathbf{3}} + H_{\mathbf{3}^*}$ and $h_I = \pm\mu_I$. In turn, μ_I are given in terms of the only available U(3) invariants h_0, $|h|^2$, and $dhhh$. Combining with the volume factor yields [23]

$$\frac{|W|}{3!} \sqrt{\left(\int J \wedge J \wedge J - 3 \int J \wedge F \wedge F \right)^2 + \left(\int F \wedge F \wedge F - 3 \int J \wedge J \wedge F \right)^2} \tag{6.20}$$

In a supersymmetric configuration the last term is absent due to (6.3) or equivalently to the condition

$$h_1 h_2 h_3 = h_1 + h_2 + h_3 \tag{6.21}$$

which can be interpreted in geometric terms as saying that $\beta_I = \arctan(h_I)$ are the three internal angles of a triangle. As a result the effective tension, which is positive as a whole may have negative contribution on some subtori. This is exactly what is needed to cancel R-R tadpole while preserving susy and without introducing lower dimensional Ω-planes [26, 21, 25]. In a sense magnetized branes behave as bound states of lower dimensional branes and anti-branes that can carry (partial) tensions and charges of different signs. The very consistency of configurations of this kind requires stabilizing the complex structure and Kähler moduli at special values depending on the magnetic fluxes. The stabilization mechanism proposed in [21] for the closed string moduli is rather simple and effective to the extent that one could neglect subtle mixings with neutral and charged (singlet) open string moduli. Imposing the holomorphy condition $F^a_{(2,0)} = 0$ for a large enough number of stacks with *oblique* magnetic fluxes fixes the complex structure moduli. Imposing the stability condition $F^a_{(1,1)} \wedge F^a_{(1,1)} \wedge F^a_{(1,1)} = 3J \wedge J \wedge F^a_{(1,1)}$ then fixes the Kähler moduli. This mechanism is particularly clear for the NS-NS moduli G_{ij}, both complex structure G_{IJ} and Kähler deformations $G_{I\bar{J}}$, which appear in the Born-Infeld action and for the R-R scalars $C_{I\bar{J}}$ associated to Kähler deformations which can combine with the anomalous magnetic U(1) gauge fields and become massive [21, 43−45]. However the R-R scalars C_{IJ} associated to complex structure deformations cannot directly mix with gauge fields whose internal field strenghts are of complex type (1,1) at the supersymmetric point. The presence of internal magnetic fluxes, however, allows all R-R fields to mix with one another and with NS-NS fields even at zero spacetime momentum. In order to show this one has to generalize the results of [27, 28], where closed string scattering amplitudes on Dp-branes were computed, to the case of our interest, i.e. D9-branes with internal magnetic fields. Setting $\alpha' = 1$ for simplicity, the relevant amplitudes are all of the form

$$\mathcal{A}(p_1, \xi_1; p_2, \xi_2) = -\frac{i}{2}\kappa\tau_p \frac{\Gamma(s)\Gamma(t)}{\Gamma(1+t+s)}\mathcal{K}(p_1, \xi_1; p_2, \xi_2) \tag{6.22}$$

where $s = p_1 D p_1$, respectively $t = (p_1 + p_2)^2$, denote the kinematic invariants in the open, respectively closed, string exchange channels, while ξ_1, ξ_2 indicate the polarizations of the external closed string states. The boundary reflection conditions are coded in the matrix D. For a Dp-brane with $F = 0$, one has $D_{\mu\nu} = \eta_{\mu\nu}$, $D_{\mu i} = D_{j\nu} = 0$, $D_{ij} = -\delta_{ij}$. Momentum conservation along the longitudinal $p + 1$ directions is understood.

We are interested in adapting the original amplitudes to the case $F \neq 0$ in the internal directions, so that $D(F) = \mathbb{1} \oplus R(F)$ and $M(F) = \mathbb{1} \otimes \Lambda(F)$, where $\Lambda(F)$ is the spinor representation of $R(F)$. We focus on the two-point amplitude mixing an internal metric fluctuation, $\phi_{ij} = \phi_{ji}$, with an internal R-R field, $\chi_{ij} = -\chi_{ji}$. Imposing spacetime momentum conservation only at the very end and slightly relaxing the mass-shell condition, by allowing some small non conserved internal momentum / winding, yields

$$\mathcal{K}^{R-R/NS-NS} = \chi^{(1)}_{ij}\phi^{(2)}_{mn}\left[i\frac{s}{2\sqrt{2}}(s+t)R^n{}_k\langle\gamma^{ij}\Lambda\gamma^k\gamma^m\rangle + i\frac{t}{(2)\sqrt{2}}t\langle\gamma^{ij}\Lambda\rangle R^{mn}\right] \tag{6.23}$$

where $\langle\cdots\rangle$ stands for a trace over spinor indices. The small momentum expansion of the Veneziano-like amplitude yields

$$\frac{\Gamma(s)\Gamma(t)}{\Gamma(1+t+s)} = \frac{1}{st} - \frac{\pi^2}{6} + \mathcal{O}(p^4) \tag{6.24}$$

that exposes massless poles in the s and t channel.

A similar analysis for the scattering of massless closed strings on Ω-planes gives a result of the form

$$\mathcal{A}(p_1, \xi_1; p_2, \xi_2) = -\frac{i}{2t(s+t)}\kappa\tau_\Omega F(1-s, t; t+1; -1)\mathcal{K}_\Omega(p_1, \xi_1; p_2, \xi_2) \tag{6.25}$$

where $F(1-s, t; t+1; -1) = t\int_0^1 dy y^{t-1}(1+y)^{s-1}$ is a hypergeometric function.

The massless t channel poles of the various amplitudes correspond to closed string massless tadpoles in the NS-NS and R-R sectors. They cancel in an anomaly free supersymmetric backgrounds [46 − 51], where the scalar fields extremize the potential, after summing the contributions of (magnetized) D-branes and Ω-planes .

The s channel poles correspond to the exchange of massless open string states that mix with the closed string states. This mixing receives no contribution from the projective plane that cannot accomodate open string channels. Indeed $F(1-s,t;t+1;-1)$ is finite for $s=0$. As argued in [21] this open-closed string mixing is responsible for lifting part of the moduli, i.e. the R-R axions $\chi_{I\bar{J}}$ of type (1,1) partecipating in the generalized GS mechanism of cancellation of the anomalous U(1)'s that schematically reads

$$\langle F^3\rangle_{1-\mathrm{loop}} - \langle F\beta\rangle_{1/2-\mathrm{loop}}\langle\beta\chi\rangle_{\mathrm{tree}}\langle\chi FF\rangle_{1/2-\mathrm{loop}} = 0 \tag{6.26}$$

where $\beta_{\mu\nu KL\bar{M}\bar{N}}$ is the R-R two-form dual (in 4-d) to the axionic R-R scalars $\chi_{I\bar{J}}$.

Even after subtracting the open string massless poles, there is a finite remnant that leads to R-R / NS-NS mixing of the schematic form

$$\mathcal{L}_{\mathrm{mix}} = \chi_{I\bar{J}}\phi_{K\bar{L}}\langle\gamma^{I\bar{J}}\Lambda\gamma^{K}\gamma_{M}\rangle R^{M\bar{L}} + \chi_{IJ}\phi_{\bar{K}\bar{L}}\langle\gamma^{IJ}\Lambda\gamma^{\bar{K}}\gamma_{M}\rangle R^{M\bar{L}}\,. \tag{6.27}$$

These include terms responsible for the mixing of the (2,0)-type R-R fields with the NS-NS (2,0) type metric fluctuations and for their mass generation. At first sight this looks puzzling[3] since a non derivative coupling of R-R scalars seems to violate invariance under their axionic shift symmetries. Moreover the standard form of the WZ couplings seems to prevent such mixings. However one should not forget that the R-R 6-form C_6 that appears in S_{WZ} is not an elementary field but rather a composite object arising from C_2 after Poincaré duality. This in turn involves the metric as required for the mixing. Another way to see this is simply solving for C_6 and working with the modified 3-form field strength for C_2, i.e. $H_3 = dC_2 + e^{\phi}\omega_3^{CS}$. The beating term $e^{-\phi}dC_2 \wedge *\omega_3^{CS}$ in the expansion of the kinetic term for C_2 arises at the correct order (disk) and once integrated by parts can produce the desired mixing with the NS-NS metric fluctuations.

A detailed resolution of the mixing requires dealing with subtle relative normalization issues among the various amplitudes and is beyond the scope of the present investigation. In specific models, such as the AM model, one can accomplish the task with some effort and motivation. Suffice it to say that the resulting masses are highly non-trivial functions of the fluxes but, being supersymmetric, should satisfy sum rules such as $Str(\mathcal{M}^2) = 0$. In principle this could be checked by computing two-point amplitudes of closed string fermions on the disk and projective plane.

Before concluding this section let us briefly mention the role of open string moduli (Wilson lines) [31] and of additional fluxes associated to metric torsion (Scherk-Schwarz deformations [52 − 54]) and to internal 3-form fluxes [37, 38]. By close analogy with the heterotic string on twisted tori [36], one can easily conclude that part of the open string

[3]M.B. would like to thank C. Angelantonj, S. Ferrara and J. F. Morales Morera for a very enlightening discussion on this topic.

Wilson line moduli are eaten by the six NS-NS graviphotons $G_{\mu i}$ very much as part of the R-R axions are eaten by the 'anomalous' U(1) vector bosons $A_{\mu a}$ in the open string spectrum. Adding metric torsion contributes extra mass terms for the six R-R graviphotons $B_{\mu i}$ in a perturbative setting. Indeed Scherk-Schwarz deformations of type I compactifications have been under active investigation for some time [53, 54]. In order to stabilize the dilaton and the R-R axion it seems however necessary to turn on internal R-R 3-form fluxes[4]. This would spoil the nice perturbative framework of the analysis performed here. In any case it would be very important to study the conditions for moduli stabilization and unbroken susy in the presence of combined constant fluxes from the three sectors of type I strings: magnetic fields (m^a_{ij}), NS-NS torsion à la Scherk-Schwarz (γ^k_{ij}) and R-R fluxes (β_{ijk}). We cannot help writing down the resulting non-abelian algebra satisfied by the generators of internal translations P_i, Cartan subalgebra of the Chan-Paton group T_a and axionic shifts W^i [36]

$$\begin{aligned} &[P_i, P_j] = 2im^a_{ij}T_a + 2i\gamma^k_{ij}P_k - 3i\beta_{ijk}W^k \\ &[P_i, T^a] = -2im^a_{ij}W^jT_a\,, \qquad \left[P_i, W^k\right] = -2i\gamma^k_{ij}W^j \end{aligned} \tag{6.28}$$

$$\left[W^i, W^k\right] = 0\,, \qquad [T_a, W^k] = 0\,, \qquad [T_a, T_b] = 0\,. \tag{6.29}$$

The structure constants are constrained by Jacobi identities

$$\begin{aligned} \gamma^l_{n[i}\gamma^n_{kj]} &= 0\,, & \gamma^k_{kj} &= 0 \\ m_{a[ij}m^a_{kl]} &= 3\beta_{n[ij}\gamma^n_{kl]}\,, & m^a_{n[k}\gamma^n_{ij]} &= 0 \end{aligned} \tag{6.30}$$

which admit a geometric interpretation in terms of Bianchi identities for the internal p-form fields.

7. Conclusions and perspectives

The results presented above should have clarified several features of the dynamics of open strings coupled to *oblique* magnetic fields. In turn these can give a host of new possibilities for symmetry breaking, moduli stabilization and chiral asymmetry even in toroidal compactifications while preserving some residual supersymmetry. Clearly these notes were aimed at a general analysis rather then the construction of specific models where all or some of the above effects are present. Some explicit models have been very recently discussed in [25, 55], where a puzzle first raised in [56] has been successfully solved along the lines of [26] but only "parallel" magnetic field have been considered. Further details can be found in [30]. More phenomenological aspects of *oblique* internal magnetic fields are under investigation [57].

Before concluding, let us briefly discuss toroidal orbifolds and models which are based on rational conformal field theories [46, 47, 50], such as Gepner models [49, 51]. As shown in [6] for the case of the NS penta-brane, governed by the product of an $SU(2)_k$ WZW model

[4]M. B. would like to thank I. Antoniadis for a discussion on this point prior to the appearance of [55].

and a non-compact free boson with a background charge Q_k, turning on magnetic fields corresponds to changing the phase of some boundary reflection coefficients in the transverse closed string channel. This in turn generates "twisted" representations in the direct open string channel, whose modes are shifted by an *a priori* rather arbitrary amount. In the case studied in [6] supersymmetry is completely broken by the introduction of magnetic fluxes but in more general cases where the chiral algebra on the worldsheet admits a spectral flow it should be possible to tune them in such a way as to preserve some supersymmetry [23, 21].

As pointed out above, the non-linearity of the abelian Born-Infeld action allow for more general possibilities of non BPS brane configurations that accomodate magnetic fields stisfying a generalized non-linear (anti) self-duality condition of the type $H_r = 1/H_s$ where r and s label two (1,1) forms. This kind of branes may offer further opportunities for phenomenologically viable type I models or their T-dual intersecting brane models [58, 59].

It is important to stress that contrary to the case of combined closed string NS-NS and R-R fluxes [37, 38], abelian magnetic fields coupled to the ends of open strings are completely under control in string perturbation theory. One can straightforwardly compute the spectrum and interactions. In the present notes we have focussed on how to derive the spectrum, in terms of magnetic shifts, multiplicities and rescalings of the internal KK momenta, but it is rather easy to generalize our analyis and extract some relevant low-energy couplings in addition to the scalar potential [30]. Very much as in intersecting brane models which are T-dual to type I compactifications with "parallel" magnetic fluxes [60], tree-level gauge couplings are simply given by

$$\frac{1}{g_a^2} = |W_a| e^{-\phi} \sqrt{\det(G + F_a)} \tag{7.1}$$

that further simplify at supersymmetric extrema. Notice the role of the wrapping factors $|W_a| = \det(W_a)^i_\alpha$ in the relative rescaling of the gauge couplings wrt to one another. Due to R-R tadpole cancellation $|W_a|$ are also important in reducing the rank of the gauge group. The one-loop running can be determined turning on (small) magnetic fields along the non-compact spacetime directions as in [60].

Another important aspect that has been investigated in [21, 55] is the possibility of accomodating some large extra dimensions [62]. We hope to get back to this point in [57] together with a preliminary analysis of Yukawa couplings for models with *oblique* internal magnetic fields extending the analysis of the "parallel" case in [64]. This should help clarifying the mixing among open and closed string moduli, a crucial issue for the problem of complete vacuum stabilization that has been only partially addressed here.

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A. The AM model

In this appendix we display the rotation matrices R_a and list the eigenvalues of $R_a^{q_a} R_b^{q_b}$ for the AM toroidal model constructed by Antoniadis and Maillard [21]. It consists of 9 stacks of magnetized D9-branes and one stack of unmagnetized ones. An explicit realization of moduli stabilization through "tangled" U(1) fluxes is shown. In table 1, we list the number of branes and the numerical value of magnetic fields at the stable supersymmetric point.

Stacks	magnetic and wrapping numbers	magnetic fields
$N_0 = 3$	$(m_{ij}, n_{ij}) = (0,1)$	$H_{ij} = 0$
$N_1 = 1$	$(m_{x_1y_2}, n_{x_1y_2}) = (1,-1)$	$H_{x_1y_2} = -\sqrt{2}/2$
	$(m_{x_2y_1}, n_{x_2y_1}) = (1,-1)$	$H_{x_2y_1} = -\sqrt{2}/2$
	$(m_{x_3y_3}, n_{x_3y_3}) = (0,-1)$	$H_{x_3y_3} = 0$
$N_2 = 1$	$(m_{x_1y_3}, n_{x_1y_3}) = (1,-1)$	$H_{x_1y_3} = -1/2$
	$(m_{x_3y_1}, n_{x_3y_1}) = (1,-1)$	$H_{x_3y_1} = -1/2$
	$(m_{x_2y_2}, n_{x_2y_2}) = (0,-1)$	$H_{x_2y_2} = 0$
$N_3 = 1$	$(m_{x_1x_2}, n_{x_1x_2}) = (1,-1)$	$H_{x_1x_2} = -\sqrt{2}/2$
	$(m_{y_1y_2}, n_{y_1y_2}) = (1,-1)$	$H_{y_1y_2} = -\sqrt{2}/2$
	$(m_{x_3y_3}, n_{x_3y_3}) = (0,-1)$	$H_{x_3y_3} = 0$
$N_4 = 2$	$(m_{x_1y_1}, n_{x_1y_1}) = (0,-1)$	$H_{x_1y_1} = 0$
	$(m_{x_2x_3}, n_{x_2x_3}) = (1,-1)$	$H_{x_2x_3} = -1$
	$(m_{y_2y_3}, n_{y_2y_3}) = (1,-1)$	$H_{y_2y_3} = -1$
$N_5 = 1$	$(m_{x_1x_3}, n_{x_1x_3}) = (1,-1)$	$H_{x_1x_3} = -1/2$
	$(m_{x_2y_2}, n_{x_2y_2}) = (0,-1)$	$H_{x_2y_2} = 0$
	$(m_{y_1y_3}, n_{y_1y_3}) = (1,-1)$	$H_{y_1y_3} = -1/2$
$N_6 = 2$	$(m_{x_1y_1}, n_{x_1y_1}) = (0,-1)$	$H_{x_1y_1} = 0$
	$(m_{x_2y_3}, n_{x_2y_3}) = (1,-1)$	$H_{x_2y_3} = -1$
	$(m_{x_3y_2}, n_{x_3y_2}) = (1,-1)$	$H_{x_3y_2} = -1$
$N_7 = 2$	$(m_{x_1y_1}, n_{x_1y_1}) = (-2,1)$	$H_{x_1y_1} = -\frac{1}{\sqrt{2}}$
	$(m_{x_2y_2}, n_{x_2y_2}) = (0,1)$	$H_{x_2y_2} = 0$
	$(m_{x_3y_3}, n_{x_3y_3}) = (1,1)$	$H_{x_3y_3} = \frac{1}{\sqrt{2}}$
$N_8 = 2$	$(m_{x_1y_1}, n_{x_1y_1}) = (-1,1)$	$H_{x_1y_1} = -\frac{1}{2\sqrt{2}}$
	$(m_{x_2y_2}, n_{x_2y_2}) = (1,1)$	$H_{x_2y_2} = \sqrt{2}$
	$(m_{x_3y_3}, n_{x_3y_3}) = (-1,1)$	$H_{x_3y_3} = -\frac{1}{\sqrt{2}}$
$N_9 = 1$	$(m_{x_1y_1}, n_{x_1y_1}) = (0,1)$	$H_{x_1y_1} = 0$
	$(m_{x_2y_2}, n_{x_2y_2}) = (-1,1)$	$H_{x_2y_2} = -\sqrt{2}$
	$(m_{x_3y_3}, n_{x_3y_3}) = (2,1)$	$H_{x_3y_3} = \sqrt{2}$

Table 1: Numbers of branes and numerical values of the magnetic fields at the stable supersymmetric point of the AM model.

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Notice that all wrapping numbers are equal to ± 1.

The first six stacks of magnetized branes, each preserving $\mathcal{N}=2$ susy, share a common $\mathcal{N}=1$ susy. The supersymmetry conditions fix the nine deformations (6 NS-NS, 3 R-R) of the complex structure to be diagonal and purely imaginary, $\tau_{IJ}=i\delta_{IJ}$. In addition there are three stacks of magnetized branes, two of which preserve $\mathcal{N}=2$ susy, while the remaining one preserves the $\mathcal{N}=1$ susy shared by all the branes. The stability conditions, trivially satisfied by the first six stacks, fix the Kähler structure when imposed on the latter three stacks.

The resulting Kähler form reads $J=J_{(1)}+J_{(2)}+J_{(3)}$ with

$$J_{(1)} = 4\pi^2 2\sqrt{2}\alpha' \frac{dz_1 \wedge d\bar{z}_1}{2i} \tag{A.1}$$

$$J_{(2)} = \frac{4\pi^2}{\sqrt{2}}\alpha' \frac{dz_2 \wedge d\bar{z}_2}{2i} \tag{A.2}$$

$$J_{(3)} = 4\pi^2 \sqrt{2}\alpha' \frac{dz_3 \wedge d\bar{z}_3}{2i} \tag{A.3}$$

i.e. radii $r_1=r_2=2^{3/4}\sqrt{\alpha'}, r_3=r_4=2^{-1/4}\sqrt{\alpha'}, r_5=r_6=2^{1/4}\sqrt{\alpha'}$. With the above supersymmetric choice of fluxes, we computed the ortogonal 6×6 matrix, R_a, for each stack of branes. They read

$$R_1^{\pm 1} = \begin{pmatrix} \frac{1}{3}\mathbb{1} & \pm\frac{2\sqrt{2}}{3}i\sigma_2 & 0 \\ \pm\frac{2\sqrt{2}}{3}i\sigma_2 & \frac{1}{3}\mathbb{1} & 0 \\ 0 & 0 & \mathbb{1} \end{pmatrix} \qquad R_3^{\pm 1} = \begin{pmatrix} \frac{1}{3}\mathbb{1} & \pm\frac{2\sqrt{2}}{3}\mathbb{1} & 0 \\ \mp\frac{2\sqrt{2}}{3}\mathbb{1} & \frac{1}{3}\mathbb{1} & 0 \\ 0 & 0 & \mathbb{1} \end{pmatrix}$$

$$R_2^{\pm 1} = \begin{pmatrix} \frac{3}{5}\mathbb{1} & 0 & \pm\frac{4}{5}i\sigma_2 \\ 0 & \mathbb{1} & 0 \\ \pm\frac{4}{5}i\sigma_2 & 0 & \frac{3}{5}\mathbb{1} \end{pmatrix} \qquad R_5^{\pm 1} = \begin{pmatrix} \frac{3}{5}\mathbb{1} & 0 & \pm\frac{4}{5}\mathbb{1} \\ 0 & \mathbb{1} & 0 \\ \mp\frac{4}{5}\mathbb{1} & 0 & \frac{3}{5}\mathbb{1} \end{pmatrix}$$

$$R_4^{\pm 1} = \begin{pmatrix} \mathbb{1} & 0 & 0 \\ 0 & 0 & \pm\mathbb{1} \\ 0 & \mp\mathbb{1} & 0 \end{pmatrix} \qquad R_6^{\pm 1} = \begin{pmatrix} \mathbb{1} & 0 & 0 \\ 0 & 0 & \pm i\sigma_2 \\ 0 & \pm i\sigma_2 & 0 \end{pmatrix}$$

$$R_7^{\pm 1} = \begin{pmatrix} \frac{1}{3}\mathbb{1} \pm \frac{2\sqrt{2}}{3}i\sigma_2 & 0 & 0 \\ 0 & \mathbb{1} & 0 \\ 0 & 0 & \frac{1}{3}\mathbb{1} \mp \frac{2\sqrt{2}}{3}i\sigma_2 \end{pmatrix}$$

$$R_8^{\pm 1} = \begin{pmatrix} \frac{7}{9}\mathbb{1} \pm \frac{4\sqrt{2}}{9}i\sigma_2 & 0 & 0 \\ 0 & -\frac{1}{3}\mathbb{1} \mp \frac{2\sqrt{2}}{3}i\sigma_2 & 0 \\ 0 & 0 & \frac{1}{3}\mathbb{1} \pm \frac{2\sqrt{2}}{3}i\sigma_2 \end{pmatrix}$$

$$R_9^{\pm 1} = \begin{pmatrix} \mathbb{1} & 0 & 0 \\ 0 & -\frac{1}{3}\mathbb{1} \pm \frac{2\sqrt{2}}{3} i\sigma_2 & 0 \\ 0 & 0 & -\frac{1}{3}\mathbb{1} \mp \frac{2\sqrt{2}}{3} i\sigma_2 \end{pmatrix}$$

Switching to complex coordinates $z^I = x^I + iy^I$ the above matrices become block diagonal

$$R_a^{\pm 1} = \begin{pmatrix} U_a^{\pm 1} & 0 \\ 0 & U_a^{*\,\pm 1} \end{pmatrix} \tag{A.4}$$

where the sub-blocks U_a are 3×3 complex matrices belonging to SU(3) $\subset$ SU(4). They read

$$U_1^{\pm 1} = \begin{pmatrix} \frac{1}{3} & \mp\frac{2\sqrt{2}}{3} i & 0 \\ \mp\frac{2\sqrt{2}}{3} i & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad U_3^{\pm 1} = \begin{pmatrix} \frac{1}{3} & \pm\frac{2\sqrt{2}}{3} & 0 \\ \mp\frac{2\sqrt{2}}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U_2^{\pm 1} = \begin{pmatrix} \frac{3}{5} & 0 & \mp\frac{4}{5} i \\ 0 & 1 & 0 \\ \mp\frac{4}{5} i & 0 & \frac{3}{5} \end{pmatrix} \qquad U_5^{\pm 1} = \begin{pmatrix} \frac{3}{5} & 0 & \pm\frac{4}{5} \\ 0 & 1 & 0 \\ \mp\frac{4}{5} & 0 & \frac{3}{5} \end{pmatrix}$$

$$U_4^{\pm 1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \pm 1 \\ 0 & \mp 1 & 0 \end{pmatrix} \qquad U_6^{\pm 1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \mp i \\ 0 & \mp i & 0 \end{pmatrix}$$

$$U_7^{\pm 1} = \begin{pmatrix} \frac{1}{3} \mp \frac{2\sqrt{2}}{3} i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \pm \frac{2\sqrt{2}}{3} i \end{pmatrix}$$

$$U_8^{\pm 1} = \begin{pmatrix} \frac{7}{9} \mp \frac{4\sqrt{2}}{9} i & 0 & 0 \\ 0 & -\frac{1}{3} \pm \frac{2\sqrt{2}}{3} i & 0 \\ 0 & 0 & \frac{1}{3} \mp \frac{2\sqrt{2}}{3} i \end{pmatrix}$$

$$U_9^{\pm 1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{3} \mp \frac{2\sqrt{2}}{3} i & 0 \\ 0 & 0 & -\frac{1}{3} \pm \frac{2\sqrt{2}}{3} i \end{pmatrix}$$

Except for the stacks labelled by $a = 8$, all the submatrices have, at least, one unit eigenvalue (in fact three for $a = 0$). The associated eigenvector defines a (complex) unmagnetized direction with standard KK momenta. The other two eigenvalues are complex conjugate.

As usual the open string states can be grouped into three main sectors: neutral, singly and doubly charged ones. With an abuse of notation we will often call magnetic shifts the angles $\beta = \pi\epsilon$.

The uncharged sector comprise open strings stretching either between two unmagnetized branes and carrying standard KK momenta or strings starting and ending on the same stack of magnetized branes ($q_a = -q_b$)and carrying reshuffled KK momenta and windings.

In the singly charged sector, one endpoint of the string is on the N_0 stack, the other is on one of the stack of magnetized branes, N_a, or their images under Ω, N_a^*. The frequencies of the oscillator modes are shifted by

$$\beta_{0a}^I = \frac{1}{2\pi}\arg(\rho_a^I)\,. \tag{A.5}$$

Open strings in this sector appear with a multiplicity due to the degeneracy of the Landau levels. For $a = 8$, this is given by the product of the magnetic numbers $\prod_I m_a^I$, in all other cases it is reduced to $\prod_{I\neq u} m_a^I$ for the presence of some unmagnetized plane, generically indicated by u.

Charged strings, starting on magnetized stack and arriving at its images are doubly charged ($q_a = q_b = \pm 1$). The frequencies shift is

$$\beta_{aa}^I = 2\beta_{a0}^I = \frac{1}{\pi}\arg(\rho_a^I) \tag{A.6}$$

and the multiplicities are $\prod_I(2m_8^I)$ for $a = 8$ or $\prod_{I\neq u}(2m_a^I)$ for all $a \neq 8$.

sectors	eigenvalues	unmagnetized directions
$U_1^{q_a}U_3^{q_b}$	$\rho = 1 \qquad \rho^\pm = (1 \pm 4i\sqrt{5})/9$	$(0,0,1)$
$U_2^{q_a}U_5^{q_b}$	$\rho = 1 \qquad \rho^\pm = (9 \pm 4i\sqrt{34})/25$	$(0,1,0)$
$U_4^{q_a}U_6^{q_b}$	$\rho = 1 \qquad \rho^\pm = \pm i$	$(1,0,0)$
$U_7^{q_a}U_8^{q_b}$	$\rho_1 = -[-7 + 16q_aq_b + 2\sqrt{2}i(2q_b + 7q_a)]/27$ $\rho_2 = [1 + 8q_aq_b + 2\sqrt{2}i(q_a - q_b)]/9$ $\rho_3 = -(1 - 2i\sqrt{2}q_b)/3$	 $(0,0,1)$ for $q_a = q_b$ none for $q_a = -q_b$
$U_7^{q_a}U_9^{q_b}$	$\rho_1 = -\frac{1}{3}(1 + 2i\sqrt{2}q_b)$ $\rho_2 = -[1 + 8q_aq_b + 2\sqrt{2}i(q_a - q_b)]/9$ $\rho_3 = \frac{1}{3}(1 - 2i\sqrt{2}q_a)$	none
$U_8^{q_a}U_9^{q_b}$	$\rho_1 = \frac{1}{9}(7 - 4i\sqrt{2}q_a)$ $\rho_2 = [1 + 8q_aq_b - 2\sqrt{2}i(q_a - q_b)]/9$ $\rho_3 = [-1 + 8q_aq_b + 2\sqrt{2}i(q_a + q_b)]/9$	 $(0,1,0)$ for $q_a = q_b$ none for $q_a = -q_b$

Table 2: In the three sectors [1,3],[2,5],[4,6] diagonalization of $R_a^{q_a}R_b^{q_b}$ yields one $\rho_{ab}^u = 1$ and two complex conjugate pairs determined by $\tan(2\beta_{ab}^I) = \pm\sqrt{t_a^2 + t_b^2 + t_a^2t_b^2}$ for $I \neq u$, where $t_{a/b} = \tan(\beta_{a/b}^I)$. In the three sectors [7,8],[7,9],[8,9], due to the diagonal form of the submatrices $U_i^{q_a}$, the magnetic shifts are simply $\beta_{ab}^I = \pm\beta_a^I \pm \beta_b^I$.

sectors	eigenvalues	unmagnetized directions
$U_4^{q_a}U_1^{q_b}$	$\rho=1 \qquad \rho^{\pm}=-(1\pm i2\sqrt{2})/3$	$(-iq_aq_b\sqrt{2},q_a,1)$
$U_4^{q_a}U_3^{q_b}$	$\rho=1 \qquad \rho^{\pm}=-(1\pm i2\sqrt{2})/3$	$(q_aq_b\sqrt{2},q_a,1)$
$U_6^{q_a}U_1^{q_b}$	$\rho=1 \qquad \rho^{\pm}=-(1\pm i2\sqrt{2})/3$	$(-q_aq_b\sqrt{2},-iq_a,1)$
$U_6^{q_a}U_3^{q_b}$	$\rho=1 \qquad \rho^{\pm}=-(1\pm i2\sqrt{2})/3$	$(-iq_aq_b\sqrt{2},-iq_a,1)$
$U_2^{q_a}U_4^{q_b}$	$\rho=1 \qquad \rho^{\pm}=-(1\pm i2\sqrt{6})/5$	$(2iq_a,q_b,1)$
$U_2^{q_a}U_6^{q_b}$	$\rho=1 \qquad \rho^{\pm}=-(1\pm i2\sqrt{6})/5$	$(2iq_a,-iq_b,1)$
$U_5^{q_a}U_4^{q_b}$	$\rho=1 \qquad \rho^{\pm}=-(1\pm i2\sqrt{6})/5$	$(-2q_a,q_b,1)$
$U_5^{q_a}U_6^{q_b}$	$\rho=1 \qquad \rho^{\pm}=-(1\pm i2\sqrt{6})/5$	$(-2q_a,-iq_b,1)$
$U_1^{q_a}U_2^{q_b}$	$\rho=1 \qquad \rho^{\pm}=(1\pm i4\sqrt{14})/15$	$(iq_b/2,-q_aq_b/\sqrt{2},1)$
$U_1^{q_a}U_5^{q_b}$	$\rho=1 \qquad \rho^{\pm}=(1\pm i4\sqrt{14})/15$	$(-q_b/2,-iq_aq_b/\sqrt{2},1)$
$U_3^{q_a}U_2^{q_b}$	$\rho=1 \qquad \rho^{\pm}=(1\pm i4\sqrt{14})/15$	$(iq_b/2,iq_aq_b/\sqrt{2},1)$
$U_3^{q_a}U_5^{q_b}$	$\rho=1 \qquad \rho^{\pm}=(1\pm i4\sqrt{14})/15$	$(-q_b/2,-q_aq_b/\sqrt{2},1)$

Table 3: The twelve sectors [13]-[25], [25]-[46] and [46]-[13] all have one unit eigenvalue (one complex unmagnetized direction/plane) and a complex conjugate pair, $\rho_{ab}^{\pm}$.

sectors	eigenvalues	unmagnetized directions
$U_7^{q_a}U_{1,3}^{q_b}$	$\rho_{1,2}=\pm(1+2i\sqrt{2}q_a)/3$ $\rho_3=(7+4iq_a\sqrt{2})/9$	none
$U_7^{q_a}U_{2,5}^{q_b}$	$\rho_1=1 \qquad \rho_{2,3}=(1\pm 2i\sqrt{6})/5$	(0,1,0)
$U_7^{q_a}U_{4,6}^{q_b}$	$\rho_1=(1-2i\sqrt{2}q_a)/3$ $\rho_{2,3}=\pm(i\sqrt{2}-q_a)/\sqrt{3}$	none
$U_8^{q_a}U_{1,3}^{q_b}$	$\rho_1=(1-2i\sqrt{2}q_a)/3$ $\rho_{2,3}=[2\mp q_a\sqrt{241}+i\sqrt{2}(q_a\pm\sqrt{241})]/27$	none
$U_8^{q_a}U_{2,5}^{q_b}$	$\rho_1=1$ $\rho_{2,3}=-(1\pm 2i\sqrt{2})/3$	$(-q_b(\sqrt{2}i+5q_a),0,3\sqrt{2})$
$U_8^{q_a}U_{4,6}^{q_b}$	$\rho_1=(7-4i\sqrt{2}q_a)/9$ $\rho_{2,3}=\pm(2\sqrt{2}i-q_a)/3$	none
$U_9^{q_a}U_{1,3}^{q_b}$	$\rho_1=-(1-2\sqrt{2}iq_a)/3$ $\rho_{2,3}=[1\pm\sqrt{52}-i\sqrt{2}q_a(1\mp\sqrt{13})]/9$	none
$U_9^{q_a}U_{2,5}^{q_b}$	$\rho_1=-(1+2\sqrt{2}iq_a)/3$ $\rho_{2,3}=[3\pm 2\sqrt{33}+iq_a\sqrt{6}(\sqrt{3}\mp\sqrt{11})]/15$	none
$U_9^{q_a}U_{4,6}^{q_b}$	$\rho_1=1 \qquad \rho_{2,3}=\pm i$	$(1,0,0)$

Table 4: The remaining 18 sectors given by [789]-[123456] can have common unmagnetized planes and frequencies given by $\beta^I_{Lab}=\beta^I_{La}$ and $\tan(2\beta^I_{Rab})=\pm\sqrt{t^2_{aR}+t^2_{bR}+t^2_{aR}t^2_{baR}}$ for $I\neq u$.

The situation becomes more involved when strings, starting on magnetized stack of branes (say, N_a) and ending on a different stack (labelled by N_b^*, or its image), are taken into account. In all, there are 36 distinct [a,b] sectors. The results are listed in tables 2, 3 and 4.

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